

ANALYSIS OF THE DECAYS $\psi' \rightarrow J/\psi \pi^+ \pi^-$ AND $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ WITH THE HEAVY QUARK SYMMETRY

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Abstract

In this article, we study the decays $\psi' \rightarrow J/\psi \pi^+ \pi^-$ and $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ by taking into account the chiral symmetry breaking effects, the final-state interactions and the heavy quark symmetry. We can confront the predictions of the $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ decay width and differential decay width with the experimental data in the future, and obtain powerful constraints on the chiral breaking effects and the final-state interactions, and test the heavy quark symmetry.

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Key words: ψ' , J/ψ , η'_c , η_c

1 Introduction

Hadronic transitions among the charmonium and bottomonium states $\psi(mS) \rightarrow \psi(nS) \pi^+ \pi^-$ and $\Upsilon(mS) \rightarrow \Upsilon(nS) \pi^+ \pi^-$ are of particular interesting for studying the dynamics of both the heavy quarkonia and the light mesons. Such processes are usually calculated with the multipole expansion in QCD, where the heavy quarkonia are considered as the compact and nonrelativistic objects and emit soft gluons which hadronize into the light meson or light meson pair [1]. The amplitudes can be factorized into the heavy quarkonium part and the light meson part. The former part depends on the dynamics of the quarkonium and should preserve the heavy quark symmetry, while the latter part depends on the chiral dynamics and should obey the chiral symmetry [2, 3, 4].

In Ref.[5], Mannel and Urech construct an effective Lagrangian for the hadronic decays $\psi' \rightarrow J/\psi \pi^+ \pi^-$ and $\Upsilon' \rightarrow \Upsilon \pi^+ \pi^-$ based on the heavy quark symmetry and the chiral symmetry, and obtain reasonable values for the coupling constants by fitting to the invariant $\pi^+ \pi^-$ mass distributions. In Ref.[6], Yan, Wei and Zhuang observe that there are D -wave contributions besides the S -wave contributions, although the D -wave contributions are very small. The D -wave contributions were firstly predicted by the Novikov-Shifman model based on the multipole expansion in QCD combined with the chiral symmetry, current algebra and partially conserved axial-vector current [7]. In Refs.[8, 9], Guo et al observe that the final-state interactions play an important role. In the case of the transitions $\Upsilon(3S) \rightarrow \Upsilon(1S) \pi \pi$, there is a double peak in the $\pi \pi$ invariant mass spectrum [10], we have to resort to additional assumptions to describe the experimental data, such as the relativistic corrections [3], the final-state interactions and the $f_0(600)$ resonance [11, 12], the exotic $\Upsilon \pi$ resonances [8, 11, 13], the coupled channel effects [14], the $S - D$ mixing [15], the field correlators [16], etc.

The two π transition $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ has not been observed yet. Recently, the CLEO collaboration searched for the decay $\psi' \rightarrow \gamma \eta'_c$ in a sample of 25.9 million ψ' events

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collected with the CLEO-c detector, and observed no evidence for the decays $\psi' \rightarrow \gamma\eta'_c$ and $\eta'_c \rightarrow \eta_c\pi^+\pi^-$, and set the upper limit,

$$\text{Br}(\psi' \rightarrow \gamma\eta'_c) \times \text{Br}(\eta'_c \rightarrow \eta_c\pi^+\pi^-) < 1.7 \times 10^{-4}, \quad (1)$$

at the 90% confidence level [17]. It is interesting to make predictions for the decay width and differential decay width of the process $\eta'_c \rightarrow \eta_c\pi^+\pi^-$, which may be observed at the BESIII and PANDA in the future [18, 19].

In this article, we study the decays $\psi' \rightarrow J/\psi\pi^+\pi^-$ and $\eta'_c \rightarrow \eta_c\pi^+\pi^-$ with a phenomenological Lagrangian by taking into account the chiral symmetry breaking effects, the final-state interactions and the heavy quark symmetry [5, 6, 8, 9, 20, 21].

The article is arranged as follows: we study the $\pi^+\pi^-$ transitions of the ψ' and η'_c in details based on the heavy quark symmetry in Sec.2; in Sec.3, we present the numerical results and discussions; and Sec.4 is reserved for our conclusions.

2 The $\pi^+\pi^-$ transitions of the ψ' and η'_c with the heavy quark symmetry

The charmonium states can be classified according to the notation $n^{2s+1}L_j$, where the n is the radial quantum number, the L is the orbital angular momentum, the s is the spin, and the j is the total angular momentum. They have the parity and charge conjugation $P = (-1)^{L+1}$ and $C = (-1)^{L+s}$, respectively. The states have the same radial quantum number n and orbital momentum $L = 0$ can be expressed by the superfield J [20, 21],

$$J = \frac{1 + \not{v}}{2} [\psi_\mu \gamma^\mu - \eta_c \gamma_5] \frac{1 - \not{v}}{2}, \quad (2)$$

where the v^μ denotes the four velocity associated to the superfield. We multiply the charmonium fields ψ_μ and η_c with a factor $\sqrt{M_\psi}$ and $\sqrt{M_{\eta_c}}$ respectively, and they have dimension of mass $\frac{3}{2}$. The superfields have been used to construct the phenomenological Lagrangians to study the radiative transitions, pseudoscalar meson transitions and vector meson transitions among the heavy quarkonia [20, 21, 22].

The $\pi^+\pi^-$ transitions between the m and n charmonium states can be described by the following phenomenological Lagrangian [5, 21],

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \sum_{m,n} \text{Tr} [\bar{J}(m)J(n)] \left\{ g_1(m,n) \text{Tr}[(\partial_\alpha U)(\partial^\alpha U)^\dagger] + g_2(m,n) \text{Tr}[(v \cdot \partial U)(v \cdot \partial U)^\dagger] \right. \\ & \left. + g_3(m,n) \text{Tr}[M(U + U^\dagger - 2)] \right\}, \end{aligned} \quad (3)$$

where $\bar{J} = \gamma^0 J^\dagger \gamma^0$, $M = B_0 \text{diag}\{m_u, m_d, m_s\}$ with $m_\pi^2 = 2B_0 m$, $m_K^2 = B_0(m + m_s)$, $m_\eta^2 = \frac{2}{3}B_0(m + 2m_s)$ in the isospin symmetry limit $m_u = m_d = m$, the U is a 3×3 matrix that contains the pseudoscalar Goldstone fields, and the $g_i(m, n)$ denote the coupling constants. We construct the chiral symmetry breaking term $\frac{1}{2}g_3(m, n) \text{Tr} [\bar{J}(m)J(n)] \text{Tr}[M(U + U^\dagger - 2)]$ consulting Ref.[5]. In the following, we will smear the indexes (m, n) in the coupling constants $g_i(m, n)$ for simplicity.

The tree diagram amplitudes for the $\pi^+\pi^-$ transitions of the ψ' and η'_c can be written as

$$\begin{aligned} T_{\psi' \rightarrow J/\psi \pi \pi}^0 &= -\frac{4}{f_\pi^2} [g_1 p_1 \cdot p_2 + g_2 p_1^0 p_2^0 + g_3 m_\pi^2] \sqrt{M_{\psi'} M_{J/\psi}} \epsilon^* \cdot \epsilon', \\ T_{\eta'_c \rightarrow \eta_c \pi \pi}^0 &= +\frac{4}{f_\pi^2} [g_1 p_1 \cdot p_2 + g_2 p_1^0 p_2^0 + g_3 m_\pi^2] \sqrt{M_{\eta'_c} M_{\eta_c}}, \end{aligned} \quad (4)$$

where the π decay constant $f_\pi = 92 \text{ MeV}$, the p_1 and p_2 are the four-momenta of the π^+ and π^- respectively, the p_1^0 and p_2^0 are the energies of the π^+ and π^- in the laboratory frame respectively, and the ϵ and ϵ' are the polarization vectors of the ψ' and J/ψ respectively. The p_1^0 and p_2^0 can be written as functions of the momenta of pions in the center of mass frame of the $\pi\pi$ system:

$$\begin{aligned} p_1^0 &= \frac{1}{\sqrt{1-\beta^2}} (E_1^* + |\beta| |\mathbf{p}_1^*| \cos \theta_\pi^*), \\ p_2^0 &= \frac{1}{\sqrt{1-\beta^2}} (E_1^* - |\beta| |\mathbf{p}_1^*| \cos \theta_\pi^*), \end{aligned} \quad (5)$$

where the β is the velocity of the $\pi\pi$ system in the center of mass frame of the initial particle, the $p_1^* = (E_1^*, \mathbf{p}_1^*)$ and $p_2^* = (E_2^*, \mathbf{p}_2^*)$ are the four-momenta of the π^+ and π^- in the center of mass frame of the $\pi\pi$ system respectively. The $p_1^0 p_2^0$ can be written as

$$p_1^0 p_2^0 = \frac{1}{1-\beta^2} \left[\left(E_1^{*2} - \frac{\beta^2 \mathbf{p}_1^{*2}}{3} \right) P_0(\cos \theta_\pi^*) - 2\beta^2 \mathbf{p}_1^{*2} P_2(\cos \theta_\pi^*) \right], \quad (6)$$

where $|\mathbf{p}_1^*| = |\mathbf{p}_2^*| = \sqrt{\frac{m_{\pi\pi}^2}{4} - m_\pi^2}$, the $P_0(\cos \theta_\pi^*) = 1$ and $P_2(\cos \theta_\pi^*) = \frac{1}{2}(\cos^2 \theta_\pi^* - \frac{1}{3})$ are the Legendre functions [8, 9].

In Refs.[8, 9], Guo et al retain the coupling constants g_1 and g_2 , and observe that the S -wave $\pi\pi$ final-state interactions play an important role and should be properly included. The chiral unitary theory is a suitable approach for taking into account the infinite series of the re-scattering meson loops [23]. At the lowest order, the isospin $I = 0$ kernel of the Bethe-Salpeter equation is

$$V_{\pi\pi, \pi\pi}^{I=0}(s) = -\frac{s - \frac{m_\pi^2}{2}}{f_\pi^2}, \quad (7)$$

and the D -wave final-state interactions cannot be included.

The scattering amplitudes for the decays $\psi' \rightarrow J/\psi \pi^+ \pi^-$ and $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ can be written as

$$T = T^0 + T_S^0 \cdot G(m_{\pi\pi}^2) \cdot 2T_{\pi\pi, \pi\pi}^{I=0}(m_{\pi\pi}^2), \quad (8)$$

where $2T_{\pi\pi, \pi\pi}^{I=0} = \langle \pi^+ \pi^- + \pi^- \pi^+ + \pi^0 \pi^0 | T^{I=0} | \pi^+ \pi^- \rangle$, the T_S^0 are the S -wave components of the scattering amplitudes T^0 , and the $G(p^2)$ is the two-meson loop propagator,

$$\begin{aligned} G(p^2) &= i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_\pi^2 + i\epsilon} \frac{1}{(p-q)^2 - m_\pi^2 + i\epsilon}, \\ &= \frac{1}{(4\pi)^2} \left\{ \tilde{a}(\mu) + \log \frac{m_\pi^2}{\mu^2} + \sigma \log \frac{\sigma+1}{\sigma-1} \right\}, \end{aligned} \quad (9)$$

where $p^2 = m_{\pi\pi}^2$, $\sigma = \sqrt{1 - \frac{4m_\pi^2}{m_{\pi\pi}^2}}$, and $\mu = m_\rho = 770 \text{ MeV}$. Here we take the dimensional regulation to regulate the ultraviolet divergence and introduce the subtraction constant $\tilde{a}(\mu)$ as a free parameter. The full S -wave $\pi\pi \rightarrow \pi\pi$ scattering amplitude $T_{\pi\pi,\pi\pi}^{I=0}$ can be taken as the solution of the on-shell Bethe-Salpeter equation [23],

$$T_{\pi\pi,\pi\pi}^{I=0}(m_{\pi\pi}^2) = \frac{V_{\pi\pi,\pi\pi}^{I=0}(m_{\pi\pi}^2)}{1 - G(m_{\pi\pi}^2)V_{\pi\pi,\pi\pi}^{I=0}(m_{\pi\pi}^2)}, \quad (10)$$

where we have neglected the contributions from the $K\bar{K}$ channels considering the values $M_{\psi'} - M_{J/\psi} < 2m_K$ and $M_{\eta'_c} - M_{\eta_c} < 2m_K$.

The differential decay width of the transition $\psi' \rightarrow J/\psi \pi^+ \pi^-$ can be written as

$$\frac{d\Gamma_{\psi' \rightarrow J/\psi \pi^+ \pi^-}}{dm_{\pi\pi}} = \frac{1}{(2\pi)^3 8M_{\psi'}^2} \overline{\sum} \sum |T|^2 |\mathbf{p}_1^*| p_{J/\psi} d\cos\theta_\pi^*, \quad (11)$$

where

$$p_{J/\psi} = \frac{\sqrt{[M_{\psi'}^2 - (M_{J/\psi} + m_{\pi\pi})^2][M_{\psi'}^2 - (M_{J/\psi} - m_{\pi\pi})^2]}}{2M_{\psi'}}, \quad (12)$$

the $\overline{\sum} \sum$ denotes the average over the polarization vector of the initial state ψ' and the sum over the polarization vector of the final state J/ψ . The corresponding differential decay width of the transition $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ can be obtained with a simple replacement.

3 Numerical results and discussions

The coupling constants g_1 , g_2 and g_3 and the subtraction constant $\tilde{a}(\mu)$ can be fitted to the experimental data on the transition $\psi' \rightarrow J/\psi \pi^+ \pi^-$ from the BES collaboration [24]. In Refs.[5, 6, 8, 9], the coupling constant g_3 associates with the small m_π^2 is neglected. In this article, we retain the coupling constant g_3 and fit the parameters to the experimental data in four cases: 2CC, 3CC, 2CC + FSI and 3CC + FSI, respectively, where the 2CC denotes the two coupling constants g_1 and g_2 , the 3CC denotes the three coupling constants g_1 , g_2 and g_3 , the FSI denotes the final-state interactions. In the chiral limit, the Adler zero condition can be satisfied. The numerical results are plotted as the number of events versus the $\pi\pi$ invariant momentum $m_{\pi\pi}$, see Fig.1. From the figure, we can see that retaining only the coupling constants g_1 and g_2 can lead to rather satisfactory fitting, by adding the coupling constant g_3 and final-state interactions, even better fittings can be obtained.

We normalize the BES data using the width $\Gamma_{\psi'} = 286 \text{ keV}$ and the branching ratio $\text{Br}(\psi' \rightarrow J/\psi \pi^+ \pi^-) = 33.6\%$ [25]. The numerical values of the coupling constants are shown in Table 1, where the unit of the coupling constants g_1 , g_2 and g_3 is GeV^{-1} , the subtraction constant $\tilde{a}(\mu)$ is a dimensionless quantity. From the Table, we can see that the parameters from the four cases differ from each other remarkably (or significantly), the coupling constant g_3 and the final-state interactions maybe play an important role, and we should take them into account.

Using the parameters presented in Table 1, we can obtain the decay width and the differential decay width of the transition $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ in the four cases 2CC, 3CC,

| | 2CC | 3CC | 2CC+FSI | 3CC+FSI |
|---------------------------------|------------------------|--------------------------|---------------------------|-------------------------|
| g_1 | 0.0873 ± 0.0008 | 0.1086 ± 0.0039 | 0.0586 ± 0.0088 | 0.0468 ± 0.0014 |
| g_2 | -0.0258 ± 0.0010 | -0.1814 ± 0.0247 | -0.0231 ± 0.0031 | 0.0033 ± 0.0018 |
| g_3 | | 0.5098 ± 0.0787 | | -0.0794 ± 0.0062 |
| \tilde{a} | | | -1.0661 ± 0.5755 | -1.8160 ± 0.1752 |
| $\Gamma_{\eta_c \pi \pi}$ | $229.7^{+7.4}_{-7.3}$ | $140.0^{+121.9}_{-74.0}$ | $209.1^{+123.1}_{-102.6}$ | $240.3^{+34.1}_{-33.3}$ |
| $\hat{\Gamma}_{\eta_c \pi \pi}$ | $2.39^{+0.08}_{-0.08}$ | $1.46^{+1.27}_{-0.77}$ | $2.18^{+1.28}_{-1.07}$ | $2.50^{+0.35}_{-0.35}$ |

Table 1: The parameters fitted to the experimental data on the $\psi' \rightarrow J/\psi \pi^+ \pi^-$ decay, the unit of the $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ decay width is KeV, and $\hat{\Gamma}_{\eta_c \pi \pi} = \frac{\Gamma_{\eta'_c \rightarrow \eta_c \pi \pi}}{\Gamma_{\psi' \rightarrow J/\psi \pi \pi}}$.

2CC + FSI and 3CC + FSI, which are shown in Table 1 and Fig.2, respectively. From the Fig.2, we can see that the line-shapes of the differential decay width of the transition $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ differ from each other significantly in the four cases, although those parameters can all give satisfactory descriptions of the $\psi' \rightarrow J/\psi \pi^+ \pi^-$ differential decay width.

In Ref.[26], M. B. Voloshin studies the transitions $\psi' \rightarrow J/\psi \pi^+ \pi^-$ and $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ in the framework of the multipole expansion in QCD using the current algebra and the trace anomaly in QCD, and obtain the ratio,

$$\frac{\Gamma_{\eta'_c \rightarrow \eta_c \pi \pi}}{\Gamma_{\psi' \rightarrow J/\psi \pi \pi}} = 3.5 \pm 0.5, \quad (13)$$

the lower bound is compatible with the upper bound of the present prediction $2.50^{+0.35}_{-0.35}$ in the case of the 3CC+FSI. In the cases of the 3CC and 2CC+FSI, the uncertainties of the present predictions are too large. We can confront the present predictions with the experimental data at the BESIII and PANDA in the future [18, 19], and obtain powerful constraints on the chiral breaking effects and the final-state interactions, and test the heavy quark symmetry.

4 Conclusion

In this article, we study the decays $\psi' \rightarrow J/\psi \pi^+ \pi^-$ and $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ by taking into account the chiral breaking effects, the final-state interactions, and the heavy quark symmetry. We fit the parameters to the experimental data on the $\psi' \rightarrow J/\psi \pi^+ \pi^-$ from the BES collaboration, and then take those values to calculate the decay width and differential decay width of the transition $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$, which can be confronted with the experimental data in the future, and put powerful constraints on the chiral breaking effects and the final-state interactions, and test the heavy quark symmetry.

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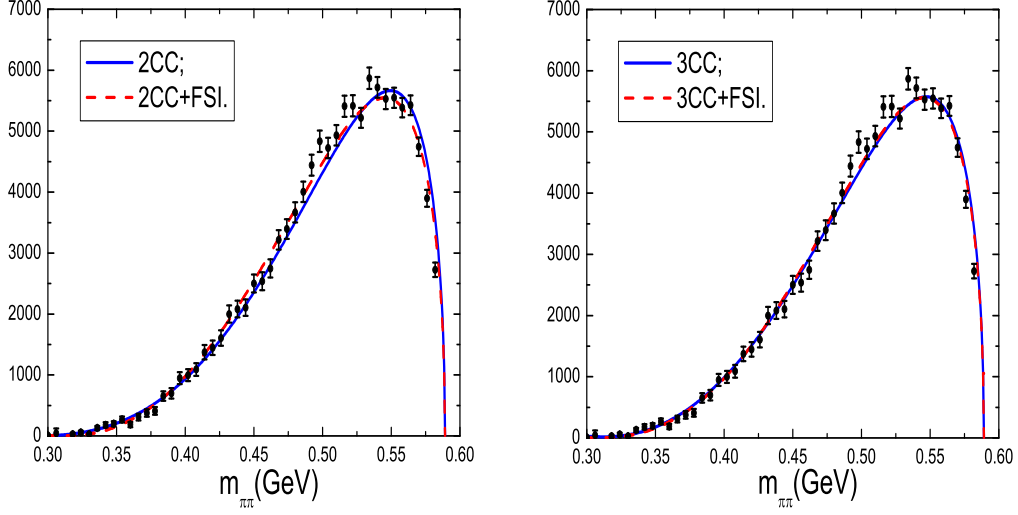


Figure 1: The number of events versus the $\pi\pi$ invariant mass distribution $m_{\pi\pi}$ in the decay $\psi' \rightarrow J/\psi \pi^+ \pi^-$, the normalization terms are not shown explicitly. The experimental data is taken from the BES collaboration.

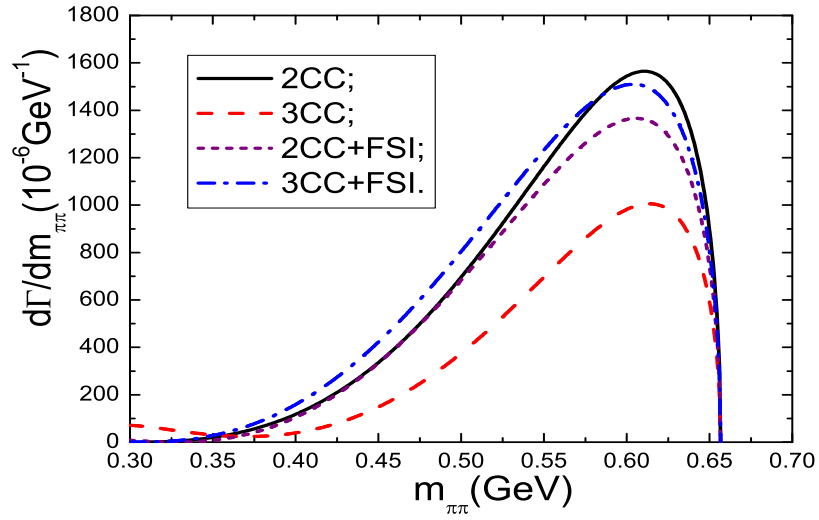


Figure 2: The differential decay width of the transition $\eta'_c \rightarrow \eta_c \pi^+ \pi^-$ versus the $\pi\pi$ invariant mass distribution $m_{\pi\pi}$.

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